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ABSTRACT

of the dissertation for the degree Doctor of Philosophy

INVESTIGATION OF THE SOLUTION OF CAUCHY AND BOUNDARY VALUE PROBLEMS FOR THE SECOND ORDER DIFFERENTIAL EQUATION WITH THREE DISCRETE DERIVATIVES

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Field of science: Mathematics
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Baku – 2022

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GENERAL CHARACTERISTICS OF THE WORK

Actuality of the research theme and degree of processing. Mathematical models of almost many natural phenomena, chemical, biological and medical phenomena are reduced to problems posed for ordinary differential equations or differential equations with partial derivatives. This is mainly the Cauchy problem, mixed and boundary problems. These questions are dealt with in courses on ordinary differential equations, equations of mathematical physics and partial differential equations. For ordinary differential equations, the Cauchy problem and boundary problems were considered, and for partial differential equations, the Cauchy problem, mixed and boundary problems.

For partial differential equations, problems were mainly considered for equations of three types, for equations of hyperbolic, parabolic and elliptic types. For equations of hyperbolic and parabolic types, the Cauchy problem and mixed problems were considered, and for equations of elliptic type - boundary value problems. The canonical form of a hyperbolic type equation is the string oscillation equation, the canonical form of a parabolic type equation is the heat equation, and the canonical form of an elliptic type equation is Laplace's equation.

For ordinary differential equations, the number of conditions in the problems under consideration must be equal to the order of the equation. In the Cauchy problem, the number of initial conditions, and in boundary problems, the number of boundary conditions must be equal to the order of the equation.

In partial differential equations, the number of initial conditions is equal to the order of the time derivative in the given equation, while the number of boundary conditions (if the number of phase variables is greater than one and the boundary is a smooth region) is equal to half the order of the derivative with respect to the phase variables. So, if the Laplace equation is a differential equation with a second-order derivative, then it is considered that one boundary condition has already been set. This is either the Dirichlet condition, or the Neumann condition, or the Poincare condition. These conditions are called local conditions. Since the biharmonic equation is an equation with a fourth-order derivative, two boundary conditions are given. Therefore, in equations of mathematical physics and equations with partial derivatives, boundary problems are mainly considered for equations with pair derivatives.

The Cauchy-Riemann equation is a first order elliptic type equation. Then in what form should the boundary value problem be considered for it in order for the problem to be a proper Fredholm-type problem.

It is shown that if we consider the Cauchy-Riemann equation in a region of a plane with a smooth boundary, then, by dividing the boundary into two regions, for the Cauchy-Riemann equation, the condition of a non-local boundary obtained by adding the values of the desired function in these parts is sufficient. That is, within a non-local boundary condition defined in this way, the problem is of the Fredholm type. In other words, the problem within the non-local boundary condition, given for the Cauchy-Riemann equation with the division of the boundary into two parts, is reduced to a system of integral equations of the Fredholm type of the second type without a singularity in the core in the normal form with respect to the boundary value of the function determined using the values obtained in the work necessary conditions.

The analysis that we go through in secondary schools and universities is an additive analysis, that is, the derivative of the sum is equal to the sum of the derivatives, and the integral of the sum is equal to the sum of the integrals. There is an additivity property here. Despite the fact that multiplicative analysis appeared in the last century (in Gantmakher's book "Theory of Matrices" the multiplicative derivative, multiplicative integral and their simple properties are given), problems for equations with a multiplicative derivative began to be considered only now.

The terms "perative derivative" and "perative integral" were given by Professor N.A. Aliyev. Since seven algebraic actions are not enough for this, it is necessary to establish a new direct action and a new reverse action.

The discrete form of additive analysis has been known for a long time. They are called equations with differences. Problems for discrete multiplicative and discrete verifiable equations were considered by N.A. Aliyev. Among the students of N.A. Aliyev: problems for differential equations with discrete additive and discrete multiplicative derivatives are found in the works of O.L. Gasanli and T.S. Mamieva, ordinary differential equations with discrete additive, discrete multiplicative and discrete equations) - in the works of A.M. Mammadzade, and, finally, problems for differential equations with ordinary and partial derivatives with these three discrete derivatives - in the works of the author of this work - V.S.Sultanova.

If we write the arbitrary equation considered here as an equation with explicit differences, we will see that the resulting equation is a very complex nonlinear equation. In all cases under consideration, the author managed to obtain an analytical expression (an explicit solution expression) for solving the problem. This determines both the relevance and the degree of study of the work.

Object and subject of research. Usually, in cases where it is difficult to study the solution of continuous problems, the problem posed is discretized, the resulting system of algebraic equations is solved, and the step considered in the solution is transferred to the limit by approaching zero, which makes it possible to arrive at a certain result for solving continuous problems.

Here our goal is to study discrete mathematical models. Basically, the Cauchy problem and boundary value problems have been considered for non-linear equations, both for ordinary differential equations and for partial differential equations.

Purpose and objectives of the study. The main goal of the dissertation work is to obtain an analytical expression for the solution of the Cauchy problem and boundary value problems for the considered very complex nonlinear equations. So, first, the general solution of the equation under consideration is determined, which consists of certain arbitrary constants or arbitrary sequences (in the case of multidimensionality). Then, arbitrary constants or sequences included in the general solution are determined using given initial or boundary conditions.

Research methods. It is known that Euler defined the function e^x invariant with respect to the additive derivative, and then the function $e^{\lambda x}$. Using this function, he algebraized a linear homogeneous differential equation with a constant coefficient. That is, he compared the differential equation with an algebraic one (characteristic equation). Then a general solution for the equation was constructed and an analytical expression for solving the problems was obtained. Here also the method already known in these differential equations for the ordinary differential equation and the partial differential equation with a discrete derivative was used.

Basic provisions for defense. The dissertation defended the following main provisions:

1. Statement of the adjoint problem and determination of the condition of self-adjointness of the considered boundary value problem for ordinary differential equations with discrete additive derivatives;

2. obtaining analytical expressions for solving Cauchy problems and boundary value problems for a first-order differential equation with discrete additive and discrete multiplicative derivatives that are linear in derivatives (in fact, non-linear);

3. obtaining analytical expressions for solving (polynomial) Cauchy problems and boundary value problems for ordinary differential equations with a second discrete derivative;

4. Obtaining analytical expressions for solving the Cauchy problem and a boundary value problem for a two-dimensional second order differential equation with discrete additive, multiplicative and differential derivatives.

Scientific novelty of the research. The following scientific novelty was obtained in the dissertation work.

1. For differential equations with a discrete additive derivative, the construction of an adjoint problem to the boundary problems of

M.A. Naimark, posed for ordinary differential equations, was carried out (for equations with differences).

2. The solution scheme given by L. Euler for ordinary differential equations was carried out for equations of the first and second order with ordinary discrete additive, multiplicative and povertive derivatives.

3. An ordinary differential equation with a discrete multiplicative and discrete first-order derivative was considered, where the study of the solution lacks the known seven algebraic actions, and therefore, it is necessary to define a new direct and a new inverse action. This action (direct action) is the action of exponentiation, its inverse action is the action of the new logarithm.

4. For the first time, in this dissertation work, the Cauchy problem and boundary value problems for multidimensional differential equations with discrete additive, multiplicative and commutative derivatives were considered. To solve the problems under consideration for two-dimensional differential equations of the second order, analytical expressions were also obtained, as well as for ordinary differential equations.

Theoretical and practical significance of the study. A dissertation work of a theoretical nature can also be used to obtain an approximate solution. These topics are taught to masters at the Department of Mathematical Methods of Applied Analysis at the Faculty of Applied Mathematics and Cybernetics of Baku State University.

Approbation and application of the work. The main provisions of the dissertation work were regularly reported at the seminars of the Department of Mathematics and Informatics of Lankaran State University, as well as at the following scientific conferences: XXXV İnternational Conference Problems of decision making under uncertainties (PDMU-2020), XXXVI İnternational Conference Problems of decision making under Uncertainties (PDMU- 2021) and the Republican scientific and practical conference of young researchers on the topic "The impact of the use of modern teaching technologies on the quality of education" (Lenkoran State University, 2019).

Personal contribution of the author. In the works (in articles and materials of conferences) the author solved the tasks set. The author is responsible for obtaining the results.

Publications of the author. The main results of the dissertation were published in 9 papers, including 6 articles, 3 conference proceedings and they are listed at the end of the abstract.

The name of the institution where the work was performed. The dissertation work was done at the Department of Mathematics and Informatics of the Lankaran State University.

The total volume of the dissertation, indicating the volume of structural units of the dissertation separately. The dissertation work consists of an introduction, three chapters, a conclusion and a list of references. Introduction - 24 pages, 31840 characters, Chapter I - 21 pages, 17704 characters, Chapter II - 25 pages, 24884 characters, Chapter III - 32 pages, 25992 characters, conclusion - 2 pages, 406 characters. The total volume of the dissertation is 110 pages, 132750 characters.

THE CONTENT OF THE WORK

In the introduction of the dissertation work, it is narrated about the way of solving discrete problems. Since these discrete problems are based on continuous problems, much space in the introduction is devoted to the stages of development of continuous problems.

Then the transition to discrete problems was carried out and the development of this area of mathematics was shown step by step.

Note that discrete events are poorly understood. Thus, well-known discrete phenomena are the determination of the general limit of the number series, the determination of the general limit of the geometric series, and the determination of the general limit of the Fibonacci sequence, showing the order of reproduction of rabbits.

The mathematical model for finding the general limit of a number series is reduced to the Cauchy problem for an ordinary differential equation with a first-order discrete additive derivative, the mathematical model for finding the general limit of a geometric series is reduced to the Cauchy problem for an ordinary differential equation with a first-order discrete multiplicative derivative, and finding the general limit of the Fibonacci sequence - to the Cauchy problem for an ordinary differential equation with a discrete second-order additive derivative.

The first chapter of the thesis work, consisting of two parts starts with

$$ly_n = y_n^{(/)} + ay_n = f_n, 0 \leq n < N \,, \tag{1}$$

$$y_N + \alpha y_0 = 0. \tag{2}$$

boundary value problem.

Adjoint problem to this boundary value problem is obtained as

$$l^* z_n = (a-1)z_n^{(/)} + az_n = g_n, 0 \leq n < N \tag{3}$$

$$\alpha z_N + z_0 = 0. \tag{4}$$

Here f_n and g_n are given sequences, a and α are given fixed numbers.

The self-adjoint condition for the given (1) to (2) problem is as follows:

$$a = 2, \alpha = 1. \tag{5}$$

The results obtained in two theorems can be given as follows.

Theorem 1. If a, α are given real numbers and f_n is a given real value sequence, then adjoint problem in (1) – (2) is in the form of (3) – (4), and $a = 2$, and $\alpha = 1$ is the self-adjoint condition of (1) – (2) problem.

Then, this chapter considers

$$ly_n \equiv y_n^{(//)} + ay_n^{(/)} + by_n = f_n \,, 0 \leq n \leq N-2, \tag{6}$$

$$\begin{cases} y_N + \alpha y_0 = 0, \\ y_{N-1} + \beta y_1 = 0, \end{cases} \tag{7}$$

boundary value problem. Adjoint problem to this problem is obtained as

$$l^* z_n \equiv (1 - a + b) z_n^{\left(\frac{-}{-}\right)} + (2b - a) z_n^{(-)} + bz_n = g_n, \quad 0 \leq n \leq N-2, \tag{8}$$

$$\begin{cases} \beta(a-2)z_N + \beta z_{N-1} + z_1 = 0, \\ \quad az_n + (a-2)z_1 + z_0 = 0, \end{cases} \tag{9}$$

and here a, b,α and β are given real constants, f_n and g_n are given real value sequences. The self-adjoint condition for (6) – (7) problem is as follows:

$$a = b = 2, \alpha = \beta = 1, \tag{10}$$

The result obtained here can be given as follows:

Theorem 2. If a, b,α and β are given real numbers and f_n is a given real value sequence, then adjoint problem to (6) – (7) is in the form of (8) – (9), and the self-adjoint condition of this problem is in the form of (10).

Here f_n and g_n sequences don't have any effect on the adjoint problem. The third chapter consisting of three parts firstly considers

$$y_n^{[/]} + ay_n^{(/)} - a^2 y_n^2 = 0, n \geq 0 \tag{11}$$

equation, here a is a given real number. The general solution of (11) equation within the condition of

$$y_n \neq C\ , \tag{12}$$

is as follows:

$$y_n = a^{2^n-1} y_0^{2^n}, n > 0, \tag{13}$$

If the initial condition of

$$y_0 = x, \tag{14}$$

is added to (11) equation, here x is a given real constant, then the solution of (11), (14) Cauchy problem is as follows:

$$y_n = a^{2^n-1} x^{2^n}, n \geq 0, \tag{15}$$

If a boundary condition is added to (11) equation

$$y_0^{\alpha} y_N^{\beta} = \gamma\ , \qquad (16),$$

here α, β and γ are given real constants. The non-single solution of this boundary value problem is as follows:

$$y_{nk} = a^{2^n-1} \left(\sqrt[\alpha+\beta 2^N]{\alpha \cdot a^{-\beta(2N-1)} e^{i\frac{2\pi k}{\alpha+\beta\cdot 2^N}}} \right)^{2^n}, \quad k \in \mathrm{Z}, \tag{17}$$

and the real solution is single and is as follows:

$$y_n = a^{2^n-1} \left(\gamma \alpha^{-\beta\left(2^{N-1}\right)}\right)^{\frac{2^n}{\alpha+\beta\cdot 2^n}}, \tag{18}$$

The result can be stated as follows.

Theorem 3. If a, α, β, γ and x are given real constants, then the general solution of the (11) equation within the (12) condition is as (13), here y_0 is arbitrary constant. In this case, the singe solution of (11), (14) Cauchy problem is as (15), and the non-single solution of the (11), (16) boundary value problem is as (17), and the single real solution is as (18).

The equation considered *in the second chapter* is a second-order discrete additive and multiplicative derivative

$$y_n^{[/]} \cdot y_n^{(/)} \left[\left(y_n^{(/)}\right)^{[/]} - \left(y_n^{[/]}\right)^{(/)} - y_n^{[/]} + 1 \right] + y_n^{(/)} = f_n y_n\ , n \geq 0\ , \tag{19}$$

equation, the general solution of which is as follows:

$$y_{2m} = y_0 \prod_{k=0}^{m-1} (1 + f_{2k})\ , m \geq 1\ , \tag{20}$$

$$y_{2m+1} = y_1 \prod_{k=0}^{m-1} (1 + f_{2k+1}) \ , m \geq 1 \ , \tag{21}$$

If

$$y_0 = \alpha, y_1 = \beta \ , \tag{22}$$

initial conditions or

$$y_0 = \alpha, y_N = \beta \ , \tag{23}$$

boundary conditions are given for this equation, then the following result can be obtained.

Theorem 4. If f_n is a given real value sequence, and α and β are given non-zero real constants, then the general solution of (19) equation is given by (20), (21), thus y_0 and y_1 are arbitrary constants. In this case, there is a single solution to the (19), (22) Cauchy problem, and this solution is as follows:

$$\begin{cases} y_{2m} = \alpha \prod_{k=0}^{m-1} (1 + f_{2k}) \ , m \geq 1, \\ y_{2m+1} = \beta \prod_{k=0}^{m-1} (1 + f_{2k+1}) \ , m \geq 1 \ , \end{cases} \tag{24}$$

if N = 2s+1, then the solution of the (19), (23) boundary value problem is as follows:

$$\begin{cases} y_{2m} = \alpha \prod_{k=0}^{m-1} (1 + f_{2k}) \ , m \geq 1, \\ y_1 = \frac{\beta}{\prod_{k=0}^{s-1} (1+f_{2k+1})}, \\ y_{2m+1} = \frac{\beta}{\prod_{k=m}^{s+1} (1+f_{2k+1})} \ , m \geq 1 \ , \end{cases} \tag{25}$$

if N = 2s, then the solution of (19) equation within

$$y_0 = \alpha, y_N = \beta \ , \tag{26}$$

boundary conditions is as follows:

$$\begin{cases} y_0 = \frac{\beta}{\prod_{k=0}^{s-1} (1+f_{2k})} \\ y_{2m+1} = \alpha \prod_{k=0}^{m-1} (1 + f_{2k+1}) \ , \ m \geq 1 \ , \\ y_{2n+1} = \frac{\beta}{\prod_{k=0}^{s-1} (1+f_{2k+1})} \ , m \geq 1 \ . \end{cases} \tag{27}$$

The last problem of the second chapter is for

$$y_n^{\{/\}^k y_n^{[/]^k}} = y_n^{k+1}, n \geq 0, \tag{28}$$

equation.

The seven algebraic operations we know are not enough to obtain a general solution to this equation. Therefore, for the general solution of equation (28), the following statement is obtained using the new direct operation "exponentiation from side" and the new inverse operation "new logarithm" operations

$$y_n = y_0^{(1+\frac{1}{k})^n}, n \geq 1, k \in N, \tag{29}$$

in which y_0 is an arbitrary constant.

If

$$y_0 = \alpha, \tag{30}$$

boundary condition is given, then the solution of (28), (30) Cauchy problem is as follows:

$$y_n = \alpha^{(1+\frac{1}{k})^n}, n \geq 1, k \in N, \tag{31}$$

If

$$y_N^\alpha - y_0^\beta = \gamma, \tag{32}$$

boundary condition is given, then the following result can be obtained:

Theorem 5. The general solution of the first order ordinary (28) equation with discrete multiplicative and poverative derivatives is as (29), since y_0 is an arbitrary constant, the solution of the (28), (30) Cauchy problem is as (31), if α, β and γ are positive numbers, the non-single solution of the (28), (32) boundary value problem is as follows:

$$y_{nm} = y_{0m}^{(1+\frac{1}{k})^n}, n \geq 1, k \in N, m \in Z, \tag{33}$$

thus, the real solution of

$$y_{0m} = y^{\frac{1}{\alpha(1+\frac{1}{k})^N+\beta}} \cdot e^{i\frac{2m\pi}{\alpha(1+\frac{1}{k})^N+\beta}}, m \in Z, \tag{34}$$

(28), (32) boundary value problem is single and is as follows:

$$y_n = y^{\frac{(1+\frac{1}{k})^n}{\alpha(1+\frac{1}{k})^N+\beta}}.$$

Finally, *the third chapter* of the thesis work devoted to the research of the solution of multidimensional problems consists of six parts, and here three Cauchy and three boundary value problems for the second order two-dimensional differential equations with discrete additive, multiplicative, and poverative derivatives have been considered.

In the first and fifth parts of this chapter, we obtain

$$D_2^{[/]} D_1^{(/)} y_{mn} = f_{mn}, m \geq 0, n \geq 0, \qquad (35)$$

second order two-dimensional differential equation with a discrete additive derivative with respect to the first variable (argument), and with a discrete multiplicative derivative with respect to the second variable within

$$y_{0n} = \alpha_{0n}, n \geq 0;\ y_{s0} = \alpha_{s0}, s \geq 0\ ; \qquad (36)$$

initial condition or by considering (35) equation within $0 \leq m < M; 0 \leq n < N$

$$\begin{cases} y_{Mn} = a y_{0n} + \varphi_n, 0 \leq n \leq N, \\ y_{mN} = b y_{m0} + \psi_m, 0 \leq m \leq M, \end{cases} \qquad (37)$$

boundary conditions.

Theorem 6. If $f_{mn}, m \geq 0, n \geq 0$, is a given real value sequence, then the general solution of the (35) equation is as follows:

$$y_{mn} = y_{0n} + \sum_{s=0}^{m-1} \left(D_1^{(/)} y_{s0}\right) \prod_{k=0}^{n-1} f_{sk}\ , m \geq 1, n \geq 1, \qquad (38)$$

thus, y_{0n} and y_{s0} are arbitrary sequences, if α_{0n} and α_{s0} are given real value sequences within (36) initial condition, then the solution of the (35), (36) Cauchy problem is as follows:

$$y_{mn} = \alpha_{0n} + \sum_{s=0}^{m-1} \left(D_1^{(/)} \alpha_{s0}\right) \prod_{k=0}^{n-1} f_{sk}\ , m \geq 1, n \geq 1, \qquad (39),$$

if $a \neq 0, a \neq 1, b \neq 0$ are given real constants, and $\varphi_n, 0 \leq n \leq N;\ \psi_m, 0 \leq m \leq M$ are given real value sequences, and

$$\prod_{k=0}^{N-1} f_{mk} \neq b, m \geq 0\,, \tag{40}$$

and

$$b\varphi_0 + \psi_M = a\psi_0 + \varphi_N, \tag{41}$$

conditions are met, then the solution of the (35), (37) boundary value problem is as follows:

$$y_{mn} = \frac{1}{a-1}\left[\sum_{s=0}^{M-1} \frac{D^{(/)}\psi_s}{\prod_{k=0}^{N-1} f_{sk} - b} \prod_{p=0}^{n-1} f_{sp} - \varphi_n\right] +$$

$$+ \sum_{s=0}^{m-1} \frac{D^{(/)}\psi_s}{\prod_{k=0}^{N-1} f_{sk} - b} \prod_{p=0}^{n-1} f_{sp}\,. \tag{42}$$

In the second and third parts of this chapter, the solution of problems have been considered for

$$D_2^{\{/\}} D_1^{(/)} u_{mn} = f_{mn}, m \geq 0, n \geq 0\,, \tag{43}$$

second order two-dimensional differential equation with a discrete additive with respect to the first variable (argument), and with a discrete poverative derivative with respect to the second variable within

$$u_{m0} = \alpha_m, u_{0n} = \beta_n, m \geq 0, n \geq 0 \tag{44}$$

initial condition or by considering (43) equation within $0 \leq m < M, 0 \leq m < N$

$$\begin{cases} u_{Mn} = au_{0n} + \varphi_n, 0 \leq n \leq N\,, \\ u_{mN} = bu_{m0} + \psi_m, 0 \ \leq m \leq M\,. \end{cases} \tag{45}$$

boundary condition. If $f_{mn}, m \geq 0, n \geq 0$, is a given real value sequence, then the general solution of the (43) equation is as follows

$$u_{mn} = u_{0n} + \sum_{s=0}^{m-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{u_{s+10}-u_{s0}}}}}}} \qquad , m \geq 1, n \geq 1\,, \tag{46}$$

thus, u_{0n} and u_{s0} are arbitrary sequences, if $\alpha_m, m \geq 0$ and $\beta_n, n \geq 0$ are given real value sequences within (44) condition, then the solution of the (43), (44) Cauchy problem is as follows:

$$u_{mn} = \beta_n + \sum_{s=0}^{m-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{\alpha_{s+1}-\alpha_s}}}}}} \quad ,m \geq 1, n \geq 1\,, \qquad (47)$$

so if $\alpha_0 = \beta_0$, a and b are given real constants, $\varphi_n, 0 \leq n \leq N, \psi_m, 0 \leq m \leq M$ are given real value sequences, then the solution of (43), (45) boundary value problem within the condition of

$$a \neq 1\,, \qquad (48)$$

and existence of

$$f_{sN-1}^{f_{sN-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{u_{s+10}-u_{s0}}}}}}} = b(u_{s+10} - u_{s0}) + \psi_{s+1} + \psi_s, \;\; s \geq 0 \quad (49)$$

equation is as (46),

$$u_{0n} = \frac{\sum_{s=0}^{M-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{u_{s+10}-u_{s0}}}}}}} - \varphi_n}{a-1} \,, n \geq 1\,, \qquad (50)$$

$(u_{s+10} - u_{s0})$ is determined from the (49) equation.

Finally, *in the fourth and sixth parts of the third chapter*,

$$D_2^{\{/\}} D_1^{[/]} u_{mn} = f_{mn}, \qquad m \geq 0, n \geq 0\,, \qquad (51)$$

second order two-dimensional differential equation with a discrete multiplicative with respect to the first variable, and with a discrete poverative derivative with respect to the second variable within

$$u_{m0} = N_m\,, m \geq 0\,;\; u_{0n} = \beta_n, n \geq 0\,;\; \alpha_0 = \beta_0\,, \qquad (52)$$

Initial condition or by considering (51) equation within $0 \leq m < M$, $0 \leq n < N$ (45) boundary condition, we obtain the following judgement.

Theorem 7. If $f_{mn}, m \geq 0\,, n \geq 0$, is a given real value sequence, then the general solution of the (51) equation is as follows:

$$u_{mn} = u_{0n} \prod_{s=0}^{m-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{\frac{u_{s+10}}{u_{s0}}}}}}}} \quad ,m \geq 1\,, n \geq 1\,, \tag{53}$$

here $u_{0n}$and u_{s0} are arbitrary sequence. If $\alpha_m, m \geq 0$ and $\beta_n, n \geq 0$ given within the (52) condition are real value sequence, and $\alpha_0 = \beta_0$, then the solution of the (51), (52) Cauchy problem is as follows:

$$u_{mn} = \beta_n \prod_{s=0}^{m-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{\frac{\alpha_{s+1}}{\alpha_s}}}}}}} \quad ,m \geq 1\,, n \geq 1\,, \tag{54}$$

If the (51) equation is considered as $0 \leq m < M, 0 \leq n < N$, in this case, if a, b of (51), (52) boundary value condition are real constants, φ_n is a given real value sequence as $0 \leq n < N\,, \psi_m\,, 0 \leq m < M$, then the solution of the (51), (45) boundary value problem within the condition of

$$\prod_{s=0}^{M-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{\frac{u_{s+10}}{u_{s0}}}}}}}} \neq a, n \geq 0, \tag{55}$$

and existence of

$$\frac{u_{s+10}}{u_{s0}} = log_{f_{s0}} log_{f_{s1}} log_{f_{s2}} \ldots log_{f_{sN-1}} \frac{u_{s+10}}{u_{s0}}\,, s \geq 0\,, \tag{56}$$

equation is given as (53), here $\frac{u_{s+10}}{u_{s0}}$ statement is given from the (56) equation, and u_{0n} is given within the condition of (55) by

$$u_{0n} = \frac{\varphi_n}{\prod_{s=0}^{M-1} f_{sn-1}^{f_{sn-2}^{\cdot^{\cdot^{\cdot^{f_{s0}^{\frac{u_{s+10}}{u_{s0}}}}}}}} - a}\,, n \geq 0, \tag{57}$$

statement.

CONCLUSION

1. For ordinary differential equations with discrete additive derivatives, the adjoint problems to the considered boundary value problems are constructed and the self-adjoint condition of the problem is defined.
2. Cauchy and boundary value problems for ordinary differential equations with discrete additive, multiplicative and poverative derivatives are considered and analytical expressions are obtained for their solution.
3. The ordinary first-order discrete differential equation is considered, and the seven algebraic operations we know are not sufficient to find a general solution. It is necessary to define a new direct operation (exponentiation from side) and a new inverse operation (new logarithm).
4. Analytical expressions are obtained for the solution of Cauchy boundary value problems for discrete multidimensional equations.

The main scientific provisions of the thesis are reflected in the following publications:

1. Решение задачи Коши и граничной задачи для уравнения дискретного аддитивного и дискретно мультипликативного первого порядка // “Müasir təlim texnologiyalarının tətbiq olunmasının təhsilin keyfiyyətinə təsiri” mövzusunda gənc tədqiqatçıların Respublika Elmi-praktik konfransı. Lənkəran Dövlət Universiteti, 2019, s. 64.
2. The adjoim problem to a boundary value problem with an additive discrete derivative // XXXV İnternational Conference Problems of decision making under uncertainties (PDMU-2020), s. 15 – 16.
3. Boundary value problem for an equation with second-order partial duscrete derivatives //XXXVI İnternational Conference Problems of decision making under Uncertainties (PDMU-2021), may 11 – 14, 2021, Didicated to 80-th anniversary of Professor MykhailoBartish, s. 105 – 106.
4. Construktion of the Adjoint problem to the discret problems for the second order equcation // Advanced Mathermatical Models&Applications. Vol. 6, № 2, 2021, s. 182 – 188.
5. Problems for the first-order differential equations with descrete additive and discrete multiplicative derivatives // Journal of Counferary applied Mathemstics. Vol. 11, № 2, 2021, s. 3 – 10.
6. Задачи Коши для уравнения второго порядка с дискретными производными // Pedaqoji Universitetinin Xəbərləri. Riyaziyyat və təbiət elmləri seriyası, 2021, № 2, s. 39 – 44.
7. Boundary-Value problem for a two-dimensional second order-type equation with discrete additive and multiplicative derivatives // EESJ (AST EUROPEAN SCIENCE JOURNAL). Vol. 1, № 4(68), 2021, s. 61 – 64.
8. Построение сопряженной задачи к граничной задаче для дискретно аддитивной производной // Pedaqoji

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9. Задача Коши для двумерного дифференциального уравнения второго порядка с дискретными мультипликативными и степенными производными // Proceeding sof the Institute of Mathematics and Mechanics. National Academy of Sciences of Azerbaijan, 2021, pp. 202 – 210.

The defense will be held on **27 September 2022** at **14^{00}** at the meeting of the Dissertation council FD 2.17 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at Baku State University.

Address: Baku city, Acad. Z. Khalilov street, 23.

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Electronic versions of dissertation and its abstract are available on the official website of the Baku State University

Abstract was sent to the required addresses on 28 july **2022**.

Signed for print: 04.07.2022
Paper format: 60 × 84 1/16
Volume: 40000
Number of hard copies: 20